$$\eta_0 H = g(\tau) + f(\tau) = -\frac{1}{\eta_r} p(\tau + \sqrt{\mu_r \epsilon_r}\, \Delta) - \frac{1}{\eta_r} p(\tau - \sqrt{\mu_r \epsilon_r}\, \Delta). \tag{A4b}$$

In addition, the total tangential electric and magnetic fields are related by:

$$E = 2g(\tau) + \eta_0 H. \tag{A5}$$

Applying the center-differencing and averaging approximations:

$$p(\tau + \sqrt{\mu_r \epsilon_r}\, \Delta) - p(\tau - \sqrt{\mu_r \epsilon_r}\, \Delta) \sim 2\Delta \sqrt{\mu_r \epsilon_r}\, p'(\tau) + \tfrac{1}{3} \Delta^3 (\mu_r \epsilon_r)^{3/2} p'''(\tau) \tag{A6a}$$

$$p(\tau + \sqrt{\mu_r \epsilon_r}\, \Delta) + p(\tau - \sqrt{\mu_r \epsilon_r}\, \Delta) \sim 2p(\tau) + \Delta^2 \mu_r \epsilon_r p''(\tau), \tag{A6b}$$

where "′" denotes derivative with respect to the argument. Substituting these back into (A4),

$$2g(\tau) = \frac{2}{\eta_r} p(\tau) + 2\Delta \sqrt{\mu_r \epsilon_r}\, p'(\tau) + \frac{\Delta^2}{\eta_r} \mu_r \epsilon_r p''(\tau) + \frac{\Delta^3}{3} (\mu_r \epsilon_r)^{3/2} p'''(\tau) \tag{A7}$$

$$\eta_0 H = -\frac{1}{\eta_r} [2p(\tau) + \Delta^2 \mu_r \epsilon_r p''(\tau)]. \tag{A8}$$

Substituting (A7) and (A8) to (A5), E is related to the first and third derivatives of $p(\tau)$. Again, using (A8) and ignoring the terms involving derivatives higher than third order by assuming slowly varying field in the time scale of Δ_τ:

$$E = -\Delta \mu_r \frac{\partial \eta_0 H}{\partial \tau} + \frac{\Delta^3 \mu_r^2 \epsilon_r}{3} \frac{\partial^3 \eta_0 H}{\partial \tau^3}. \tag{A9}$$

Using Maxwell's equations:

$$E = \Delta \mu_r \frac{\partial E}{\partial y} - \frac{\Delta^3 \mu_r^2 \epsilon_r}{3} \frac{\partial^3 E}{\partial \tau^2\, \partial y}.$$

This is what one expects if one expands, in Taylor series, the *tangent* function in (1) and converts the expanded equation to the time domain using the procedure described earlier.

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Comments on "Criteria for the Onset of Oscillation In Microwave Circuits"

Robert W. Jackson

The paper listed above[1] notes that the device reflection coefficient, $\Gamma_d(s)$, in the expression,

$$V^+ = \frac{\Gamma_d(s)}{1 - \Gamma_d(s)\Gamma_c(s)} V_i$$

represents the port reflection coefficient of a device which may result in an unstable circuit only *after* connecting it with a resonator having a reflection coefficient, $\Gamma_c(s)$. This is an important condition and is somewhat vague as worded. In order to use the Nyquist criterion to determine the stability of the device-circuit combination, Γ_d must have no right half plane poles. This amounts to insuring that the device does not oscillate into the reference impedance (50 ohms for example). If Γ_d has been determined from measurements, presumably the device is not oscillating during the measurement and therefore there are no right half plane poles.

In CAD simulations of possibly unstable circuits, the location of Γ_d poles is not always so clear. For a simple amplifier circuit such as the one described in the above referenced paper, one can assume no right half plane poles in Γ_d if the S_{11} and/or S_{22} coefficients of the FET have magnitudes less than one. To see this, consider the partial circuit formed by a 50 ohm termination on port 2 and any passive termination on port 1 If $|S_{11}| < 1$, the input termination sees a passive impedance and therefore the partial circuit is stable. Since the partial circuit is stable, Γ_d (50 ohm reference) seen looking in at port 2 has no poles in the right half plane. If, as in the amplifier example[1], Γ_d has a magnitude greater than 1, the Nyquist criterion as described can then be applied to study the stability effects of various port 2 terminations. In simulations using devices with extra feedback, oscillators for example, often the magnitudes of S_{11} and S_{22} are both greater than one and this approach breaks down.

A more generally applicable use of the Nyquist stability criterion has been known for years, but the current widespread use of microwave CAD makes it must easier to apply. As discussed in the literature [1], [2] the admittance between any two nodes in an active circuit cannot have right half plane zeros if the circuit is to be stable. If one were to apply the Nyquist test to such an admittance, the resulting Nyquist locus of points cannot encircle zero in a clockwise sense if the circuit is stable. It is trivial for modern microwave CAD programs to calculate the necessary admittances vs frequency. Polar plotting of admittances is not always available but a quick sketch is easy to do. It should be noted that the number of Nyquist encirclements only gives the *difference* between the number of right half plane zeros and poles in the admittance function. If, for example, the admittance at a particular node pair has an equal number of right half plane poles and zeros, the Nyquist plot would not encircle the origin even though the circuit is unstable. Thus a clockwise encirclement insures instability, but no encirclement does not insure stability. Since admittances at various node

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[1] R. W Jackson, *IEEE Trans. Microwave Theory Tech.*, vol. 40, no. 3, pp. 566–568, Mar. 1992.

pairs will all have the same zeros but could have different poles, one can apply the test at a number of node pairs and confidently determine, but not guarantee, that a circuit is stable. Although we have only considered admittances, the same test can be applied to impedances seen looking into the terminals formed when any loop in the circuit is broken. Again, any clockwise Nyquist encirclements of the origin mean that right half plane zeros of the impedances exist and that the circuit is unstable.

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Corrections to "Open Resonator for Precision Dielectric Measurements in the 100-GHz Band"

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Owing to an error in calculation programming, tan δ in Table II in the above paper[1] was calculated as $\phi_d = 0$ in equation (3). tan δ in Table II should be changed as follows:

TABLE II
MEASURED PERMITTIVITY AND LOSS TANGENT

Material	t (mm)	f (GHz)	Δ	ϵ_r	$\tan\delta \times 10^4$
Silica	1.46	105.3	1.00	3.799	3.0
Glass				3.800	2.8
(IR grade)	2.94	105.3	1.00	3.800	3.2
				3.800	3.3
	5.12	104.7	1.00	3.800	3.2
				3.800	3.4
MgO	1.04	92.8	1.00	9.809	0.53
				9.813	0.61
AlN	1.12	92.9	1.00	8.289	5.7
				8.290	5.6
	2.18	96.5	1.01	8.296	4.3
				8.297	4.6
BN	1.93	103.0	1.00	5.163	12
				5.163	12

[1]B. Komiyama, M. Kiyokawa, and T. Matsui, *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 10, pp. 1792–1796, Oct. 1991.